

# SUSY and Flat Direction in de Sitter Space

Masahiro Tanaka

*Department of Physics, Tohoku University, Sendai, 980-77 Japan*

## Abstract

We have found that supersymmetry (SUSY) in curved space is broken *softly*. It is also found that Pauli-Villars regularization preserves the remaining symmetry, softly broken SUSY. Using it we computed the one-loop effective potential along a (classical) flat direction in a Wess-Zumino model in de Sitter space. The analysis is relevant to the Affleck-Dine mechanism for baryogenesis. The effective potential is unbounded from below:  $V_{eff}(\phi) \rightarrow -3g^2 H^2 \phi^2 \ln \phi^2 / 16\pi^2$ , where  $\phi$  is the scalar field along the flat direction,  $g$  is a typical coupling constant, and  $H$  is the Hubble parameter. This is identical with the effective potential which is obtained by using proper-time cutoff regularization. Since proper-time cutoff regularization is exact even at the large curvature region, the effective potential possesses softly broken SUSY and reliability in the large curvature region.

# 1 Introduction

A scalar field can effectively be considered as massless, if its mass  $m$  is much smaller than the Hubble parameter  $H$ . Such a scalar field has a flat direction in the field space along which the potential does not vary.

Our interest in flat directions is motivated by their use in the Affleck-Dine mechanism [1] for baryogenesis. In their scenario, just after inflation the scalar field along a flat direction has a large expectation value of the order of the GUT (Grand Unified Theory) scale and can be associated with baryon number violating operators. The large expectation value is due to quantum fluctuation of the scalar field during inflation. At the epoch right after inflation the Hubble parameter may be of the order of the GUT scale or the intermediate scale ( $\sim 10^{11}\text{GeV}$ ). After inflation and when the Hubble parameter becomes of the order of the supersymmetry (SUSY) breaking mass  $m$  (that is of the order of the weak scale  $\sim 10^2\text{GeV}$ ), the scalar field begins to fall down along the flat direction toward a true minimum of the potential. A condensate of the scalar field (say the squark field) possesses a baryon number density which is gradually diluted by the expansion of the universe, and finally decays into lighter particles. The large initial value<sup>1</sup> mean guarantees a large baryon to photon ratio which could be larger than the observed. In their scenario it is a flat direction that gives such a large initial value.

In reality the Affleck-Dine mechanism works well in supersymmetric unified theories because a globally supersymmetric model often has (exact) flat directions<sup>2</sup> and, what is more important, the (exact) flat directions are free from quantum correction due to the non-renormalization theorem [2, 3].

With a soft SUSY breaking mass  $m$ , on the other hand, the potential along the direction receives logarithmically divergent radiative correction proportional to the soft SUSY breaking mass squared  $m^2$ . But logarithmically divergent correction is much milder than quadratically divergent. If the soft SUSY breaking mass is small, quantum correction is also small. Therefore the flat direction is still flat approximately.

Although our motivation for use of flat directions comes from cosmology, it is not known if flat directions in curved space are (effectively) flat or not including

---

<sup>1</sup> The initial conditions are set at the epoch (right) after inflation.

<sup>2</sup>If there is no constant term in a potential, (exact) flat directions are zero energy states, where SUSY is unbroken.

quantum correction. To understand it is the purpose of this paper. Hereafter we refer to flat direction at tree (classical) level as flat direction. More specifically, we evaluate the one-loop effective potential[4, 5, 6, 7] along a flat direction in de Sitter space. De Sitter space is of particular interest since the inflationary phase of the universe is described by this space. Therefore the shape of the potential along flat directions in de Sitter space is crucial to the Affleck-Dine mechanism.

In this paper we consider *renormalizable* models that reduce to global (N=1) supersymmetric models in Minkowski space limit.<sup>3</sup> Although we study *supersymmetric* models in de Sitter space, we do not know how to characterize this symmetry because SUSY in curved space is broken. We must know what kind of SUSY-like symmetry remains in de Sitter space. The calculation we perform must respect this symmetry if it is ever possible. Note that the renormalizability in curved space allows a curvature coupling, or the conformal coupling,  $-\xi\mathcal{R}\phi^2$ [5, 7, 8]. Since the scalar curvature  $\mathcal{R}$  is given by  $\mathcal{R} = -12H^2$  in de Sitter space, if  $\xi$  is of the order of unity, the direction is not flat even at tree level.

## 2 Softly Broken SUSY in Curved Space

### 2.1 Softly Broken SUSY in Curved Space

We see that global SUSY is broken softly in curved space<sup>4</sup>, e.g., de Sitter space. A theory with softly broken symmetry is characterized by the following two conditions[10]. One is that the new divergences which the theory contains other than symmetric one does all appear in the operators with dimension smaller than four. The other is that the strongest superficial divergence remains unchanged even in the theory with softly broken symmetry.

For simplicity we will treat the Wess-Zumino model with a superpotential

$$P = \frac{m}{2}\phi^2 + \frac{\sqrt{2}}{3}g\phi^3. \quad (1)$$

In curved space

$$e^{-1}\mathcal{L} = \frac{1}{2}(A\Box A + B\Box B + i\bar{\psi}\not{D}\psi + F^2 + G^2) + \frac{1}{2}\xi\mathcal{R}(A^2 + B^2)$$

---

<sup>3</sup>In Ref. [6] the one-loop effective action of N=1 pure supergravity model in de Sitter space is studied.

<sup>4</sup>In anti-de Sitter space there exists global SUSY[9] because of its maximal symmetry, but in de Sitter space this is not the fact.

$$+m\left(AF - BG + \frac{1}{2}\bar{\psi}\psi\right) + g\left[\bar{\psi}(A - \gamma_5 B)\psi + F(A^2 - B^2) - 2GAB\right] \quad (2)$$

where  $A$  is the real scalar field,  $B$  is the real pseudoscalar field ( $\phi = (A + iB)/\sqrt{2}$ ), and  $\psi$  is the Majorana spinor field.  $F$  and  $G$  are the auxiliary fields in the scalar (chiral) multiplet. We include the curvature coupling with the scalar curvature  $\mathcal{R}$ .  $m$  is the supersymmetric mass. In flat space limit this Lagrangian is invariant under the supertransformation:

$$\begin{aligned} \delta A &= -\bar{\epsilon}\psi, & \delta B &= -\bar{\epsilon}\gamma_5\psi \\ \delta F &= -i\bar{\epsilon}\mathcal{D}\psi, & \delta G &= -i\bar{\epsilon}\mathcal{D}\gamma_5\psi \\ \delta\psi &= \left[i\mathcal{D}A + F - \gamma_5(i\mathcal{D}B + G)\right]\epsilon, \end{aligned} \quad (3)$$

if we choose  $\epsilon$  constant. In curved space this is not the fact:

$$\begin{aligned} e^{-1}\delta\mathcal{L} &= \mathcal{D}_\mu\bar{\epsilon} \cdot \left[-\mathcal{D}(A - \gamma_5 B) \cdot \gamma^\mu\psi + im(A + \gamma_5 B)\gamma^\mu\psi\right] \\ &\quad - \bar{\epsilon}\xi\mathcal{R}(A + \gamma_5 B)\psi + g\mathcal{D}_\mu\bar{\epsilon} \cdot i(A^2 - B^2 + 2\gamma_5 AB)\gamma^\mu\psi \end{aligned} \quad (4)$$

$$\neq 0. \quad (5)$$

To see that SUSY is broken softly we must pick up all the divergent 1PI diagrams at one-loop order. It is done by expressing the propagators in momentum representation [11]<sup>5</sup>

$$\langle A(x)A(x') \rangle = \langle B(x)B(x') \rangle \quad (6)$$

$$= \frac{i}{\square + \xi\mathcal{R} - m^2}\delta(x, x') \quad (7)$$

$$\begin{aligned} &= i \int \frac{d^4k}{(2\pi)^4} e^{ik \cdot (x-x')} \left[ -\frac{1}{k^2 + m^2} + \left(\frac{1}{6} - \xi\right) \frac{\mathcal{R}}{(k^2 + m^2)^2} \right. \\ &\quad \left. + \frac{1}{6} \frac{\mathcal{R}}{(k^2 + m^2)^2} - \frac{2}{3} \frac{\mathcal{R}_{ab}k^a k^b}{(k^2 + m^2)^3} + \mathcal{O}(\mathcal{R}^{3/2}) \right] \end{aligned} \quad (8)$$

$$\langle \psi(x)\bar{\psi}(x') \rangle = \frac{i}{i\mathcal{D}_x + m}\delta(x, x') \quad (9)$$

$$\begin{aligned} &= i \int \frac{d^4k}{(2\pi)^4} e^{ik \cdot (x-x')} \left[ \frac{\not{k} + m}{k^2 + m^2} - \frac{\mathcal{R}}{8} \frac{\not{k}}{(k^2 + m^2)^2} \right. \\ &\quad \left. - \frac{\mathcal{R}}{12} \frac{\not{k} + m}{(k^2 + m^2)^2} + \frac{2}{3} \mathcal{R}_{ab}k^a k^b \frac{\not{k} + m}{(k^2 + m^2)^3} + \mathcal{O}(\mathcal{R}^{3/2}) \right] \end{aligned} \quad (10)$$

---

<sup>5</sup>We use Riemann normal coordinates in obtaining the expressions[12].

$$\langle A(x)F(x') \rangle = -\langle B(x)G(x') \rangle = -m\langle A(x)A(x') \rangle \quad (11)$$

$$\langle F(x)F(x') \rangle = \langle G(x)G(x') \rangle = i\delta(x, x') + m^2\langle A(x)A(x') \rangle. \quad (12)$$

We can find that one, two, and three point vertex functions are logarithmically divergent and divergence due to the curvature appears only in one and two point vertex functions. Superficial degrees of divergence is unchanged in each vertex function. From the above argument it can be deduced that the curvature acts as the soft SUSY breaking mass. See Table 1.

Dim. of operator	Degrees of divergence			
	SUSY		Broken SUSY in curved space	
1	cubic	→ nothing	cubic	→ logarithmic
2	quadratic	→ logarithmic	quadratic	→ logarithmic
3	linear	→ logarithmic	linear	→ logarithmic
4	logarithmic	→ logarithmic	logarithmic	→ logarithmic

Table 1: This table shows the structure of divergence appear in the coefficients of operators. The left hand side of each arrow is the superficial degrees of divergence and the right hand side is its actual degrees of divergence.

The reason why we can get such observations is as follows. One-loop diagrams which contribute to the one point function cancel one another in exactly supersymmetric theories while each of them diverges quadratically. In de Sitter space (or in curved space) the curvature dependent part breaks cancellation but it just gives logarithmic divergence because a propagator is expanded by power of  $\mathcal{R} \times (\text{momentum})^{-2}$  ( see Eqs. (8) and (10)). One-loop contribution to the two point vertex functions is non-vanishing even in exactly supersymmetric theories and gives logarithmic divergence. Moreover, one-loop contribution from the curvature dependent parts exists and is logarithmically divergent just like the case of one point function. The three point function diverges logarithmically even at each graph, so the curvature dependent parts are finite. A reason from another point of view is explained in Appendix B.

In this paper we consider that all the suitable regularizations for SUSY in curved space must satisfy the Ward-Takahashi identity for softly broken SUSY.

## 2.2 Ward-Takahashi Identity for Softly Broken SUSY with Pauli-Villars Regulators

We derive the Ward-Takahashi identity for softly broken SUSY. We adopt Pauli-Villars regularization as a candidate of suitable regularization for softly broken SUSY in de Sitter space. It is manifestly supersymmetric in exactly supersymmetric theories[13, 9]. To check if Pauli-Villars regularization satisfies the Ward-Takahashi identity or not we write down the total Lagrangian with the regulators. The superpotential with the regulators is as follows

$$P = \frac{1}{2} \sum_i m_i \phi_i^2 + \frac{\sqrt{2}}{3} \sum_{ijk} g_{ijk} \phi_i \phi_j \phi_k. \quad (13)$$

To regularize all the divergent diagrams the regulator fields have the same coupling constant with the physical field:

$$g_{ijk} = g. \quad (14)$$

We obtain the Lagrangian;

$$\begin{aligned} e^{-1} \mathcal{L}_{tot} = & \sum_i \frac{1}{2c_i} \left[ A_i \square A_i + B_i \square B_i + i \bar{\psi}_i \not{D} \psi_i + F_i^2 + G_i^2 + \xi \mathcal{R} (A_i^2 + B_i^2) \right. \\ & + m_i (2A_i F_i - 2B_i G_i + \bar{\psi}_i \psi_i) \left. \right] + g \sum_{ijk} \left[ \bar{\psi}_i (A_j - \gamma_5 B_j) \psi_k + F_i (A_j A_k \right. \\ & - B_j B_k) - 2G_i A_j B_k \left. \right] + \sum_i \left[ J_A^i A_i + J_B^i B_i + J_F^i F_i + J_G^i G_i + \bar{\psi}_i \eta^i \right] \\ & + \sum_i \frac{1}{c_i} \left[ \bar{K}_{\partial A}^\nu \partial_\nu A_i + \bar{K}_{\partial B}^\nu \partial_\nu B_i + m_i (\bar{K}_A A_i + \bar{K}_B B_i) + \xi (\bar{K}'_A A_i \right. \\ & \left. + \bar{K}'_B B_i) \right] \psi_i + g \sum_{ijk} \left[ \bar{K}_{A^2} A_i A_j + \bar{K}_{B^2} B_i B_j + 2\bar{K}_{AB} A_i B_j \right] \psi_k. \quad (15) \end{aligned}$$

$J$ s are the sources of the bosonic fields and  $\eta$  is the source of the fermionic field.  $\bar{K}$ s are the sources of the composite operators which appear in the first variation of the Lagrangian Eq.(4). To regularize all the divergent terms we need the Pauli-Villars constraints  $\sum_i c_i m_i^p = 0$  for  $p = 0, 1, 2$  where  $c_0 = 1$  and  $m_0 = m$ . The regulators, which have  $i = 1, 2, \dots$ , have adjustable masses which we set infinite after all the calculations.  $c_i$  is the function of  $m_i$ s.<sup>6</sup> The supertransformation of the total action

---

<sup>6</sup>We can get the explicit form of  $c_i$  as follows.  $c_i = -(m_j - m)(m_k - m)/(m_j - m_i)(m_k - m_i)$  for  $i \neq j, j \neq k$  and  $k \neq i$ . If we set  $m_1 < m_2 < m_3$ , then  $c_1, c_3 < 0$  and  $c_2 > 0$ . But we do not need the explicit forms of  $c_i$ s.

is

$$\begin{aligned}
\delta S_{tot} = & \int d^4x e \left[ \sum_i \frac{1}{c_i} \mathcal{D}_\mu \bar{\epsilon} \cdot \left\{ -\mathcal{P}(A_i - \gamma_5 B_i) \gamma^\mu \psi_i + im(A_i + \gamma_5 B_i) \gamma^\mu \psi_i \right\} \right. \\
& - \sum_i \frac{1}{c_i} \bar{\epsilon} \xi \mathcal{R}(A_i + B_i \gamma_5) \psi_i + g \sum_{ijk} \mathcal{D}_\mu \bar{\epsilon} \cdot i(A_i A_j - B_i B_j + 2\gamma_5 A_i B_j) \gamma^\mu \psi_k \\
& - \bar{\epsilon} \sum_i \left\{ \psi_i J_A^i + \gamma_5 \psi_i J_B^i + i \mathcal{P} \psi_i \cdot J_F^i + i \mathcal{P} \gamma_5 \psi_i \cdot J_G^i \right. \\
& \left. \left. + (i \not{\partial} A_i - F_i - i \not{\partial} B_i \gamma_5 + G_i \gamma_5) \eta^i \right\} + (\bar{K}\text{-terms}) \right]. \tag{16}
\end{aligned}$$

It is translated to Ward-Takahashi identity by using generating functional  $Z[J, \eta, \bar{K}]$  as follows

$$\begin{aligned}
& \int d^4x e \left[ -\mathcal{D}_\mu \bar{\epsilon} \cdot \left\{ \gamma^\nu \gamma^\mu \left( \frac{\delta}{\delta \bar{K}_{\partial A}^\nu} + \gamma_5 \frac{\delta}{\delta \bar{K}_{\partial B}^\nu} \right) - i \gamma^\mu \left( \frac{\delta}{\delta \bar{K}_A} - \gamma_5 \frac{\delta}{\delta \bar{K}_B} \right) \right\} \right. \\
& - \bar{\epsilon} \mathcal{R} \left( \frac{\delta}{\delta \bar{K}'_A} + \gamma_5 \frac{\delta}{\delta \bar{K}'_B} \right) + \mathcal{D}_\mu \bar{\epsilon} \cdot i \gamma^\mu \left( \frac{\delta}{\delta \bar{K}_{A^2}} - \frac{\delta}{\delta \bar{K}_{B^2}} - \gamma_5 \frac{\delta}{\delta \bar{K}_{AB}} \right) \\
& - \bar{\epsilon} \sum_i \left\{ J_A^i \frac{\delta}{\delta \bar{\eta}^i} + J_B^i \gamma_5 \frac{\delta}{\delta \bar{\eta}^i} + i J_F^i \mathcal{P} \frac{\delta}{\delta \bar{\eta}^i} + i J_G^i \mathcal{P} \gamma_5 \frac{\delta}{\delta \bar{\eta}^i} + i \gamma^\mu \eta^i \partial_\mu \frac{\delta}{\delta J_A^i} \right. \\
& \left. - \eta^i \frac{\delta}{\delta J_F^i} - i \gamma^\mu \gamma_5 \eta^i \partial_\mu \frac{\delta}{\delta J_B^i} + \gamma_5 \eta^i \frac{\delta}{\delta J_G^i} \right\} + (\bar{K}\text{-terms}) \Big] Z[J, \eta, \bar{K}] = 0. \tag{17}
\end{aligned}$$

Using the connected generating functional  $W = -i \ln Z$  we define the effective action as follows

$$\Gamma[\phi, \psi, \bar{K}] = W[J, \eta, \bar{K}] - \int d^4x e (J \cdot \phi + \bar{\psi} \cdot \eta). \tag{18}$$

Here we denote the bosonic classical fields by  $\phi$  and the fermionic one by  $\psi$ . Then

$$\phi(x) = \frac{\delta}{\delta J(x)} W[J, \eta, \bar{K}], \quad J(x) = -\frac{\delta}{\delta \phi(x)} \Gamma[\phi, \psi, \bar{K}] \tag{19}$$

$$\psi(x) = \frac{\delta}{\delta \bar{\eta}(x)} W[J, \eta, \bar{K}], \quad \eta(x) = -\frac{\delta}{\delta \bar{\psi}(x)} \Gamma[\phi, \psi, \bar{K}] \tag{20}$$

$$\mathcal{Q}(x) = \frac{\delta}{\delta \bar{K}(x)} W[J, \eta, \bar{K}] = \frac{\delta}{\delta \bar{K}(x)} \Gamma[\phi, \psi, \bar{K}], \tag{21}$$

where  $\mathcal{Q}(x)$  represents every composite operator. We obtain the Ward-Takahashi identity for the effective action

$$\int d^4x e \left[ -\mathcal{D}_\mu \bar{\epsilon} \cdot \left\{ \gamma^\nu \gamma^\mu \left( \frac{\delta \Gamma}{\delta \bar{K}_{\partial A}^\nu} + \gamma_5 \frac{\delta \Gamma}{\delta \bar{K}_{\partial B}^\nu} \right) - i \gamma^\mu \left( \frac{\delta \Gamma}{\delta \bar{K}_A} - \gamma_5 \frac{\delta \Gamma}{\delta \bar{K}_B} \right) \right\} \right.$$

$$\begin{aligned}
& -\bar{\epsilon}\mathcal{R}\left(\frac{\delta\Gamma}{\delta\bar{K}'_A} + \gamma_5\frac{\delta\Gamma}{\delta\bar{K}'_B}\right) + \mathcal{D}_\mu\bar{\epsilon} \cdot i\gamma^\mu\left(\frac{\delta\Gamma}{\delta\bar{K}_{A^2}} - \frac{\delta\Gamma}{\delta\bar{K}_{B^2}} - \gamma_5\frac{\delta\Gamma}{\delta\bar{K}_{AB}}\right) \\
& + \bar{\epsilon}\sum_i\left\{\frac{\delta\Gamma}{\delta A_i}\psi_i + \frac{\delta\Gamma}{\delta B_i}\gamma_5\psi_i + i\frac{\delta\Gamma}{\delta F_i}\mathcal{D}\psi_i + i\frac{\delta\Gamma}{\delta G_i}\mathcal{D}\gamma_5\psi_i + i\gamma^\mu\frac{\delta\Gamma}{\delta\psi_i}\partial_\mu A_i\right. \\
& \left. - \frac{\delta\Gamma}{\delta\bar{\psi}_i}F_i - i\gamma^\mu\gamma_5\frac{\delta\Gamma}{\delta\bar{\psi}_i}\partial_\mu B_i + \gamma_5\frac{\delta\Gamma}{\delta\bar{\psi}_i}G_i\right\} + (\bar{K}\text{-terms})\Big] = 0. \tag{22}
\end{aligned}$$

All the fields in effective actions are classical fields, although we do not make no distinction between a classical field and a quantum one in notations. In the previous subsection we found that the divergent parts of the effective action are those of one, two, and three point vertex functions. Three point function diverges only at the curvature independent part, supersymmetric one, so we do not need to check the Ward-Takahashi identity for three point functions. We do not need the  $\bar{K}$ -terms explicitly, because in deriving the Ward-Takahashi identity for n-point vertex function we do not differentiate by  $\bar{K}$  and do set them zero after the differentiations with respect to fields.

## 2.3 One Point Function

In this section we check the Ward-Takahashi identity for one point vertex function. We differentiate Eq.(22) with respect to  $\psi_j(y)$  and set all the fields and the sources of composite fields zero. We obtain

$$\begin{aligned}
& \int d^4x e\left[-\mathcal{D}_\mu\bar{\epsilon} \cdot \left\{\gamma^\nu\gamma^\mu\left(\frac{\delta^2\Gamma}{\delta\psi_j(y)\delta\bar{K}_{\partial A}^\nu} + \gamma_5\frac{\delta^2\Gamma}{\delta\psi_j(y)\delta\bar{K}_{\partial B}^\nu}\right) - i\gamma^\mu\left(\frac{\delta^2\Gamma}{\delta\psi_j(y)\delta\bar{K}_A} - \gamma_5\frac{\delta^2\Gamma}{\delta\psi_j(y)\delta\bar{K}_B}\right)\right\} - \bar{\epsilon}\mathcal{R}\left(\frac{\delta^2\Gamma}{\delta\psi_j(y)\delta\bar{K}'_A} + \gamma_5\frac{\delta^2\Gamma}{\delta\psi_j(y)\delta\bar{K}'_B}\right) + \mathcal{D}_\mu\bar{\epsilon} \cdot i\gamma^\mu \cdot \right. \\
& \left. \left(\frac{\delta^2\Gamma}{\delta\psi_j(y)\delta\bar{K}_{A^2}} - \frac{\delta^2\Gamma}{\delta\psi_j(y)\delta\bar{K}_{B^2}} - \gamma_5\frac{\delta^2\Gamma}{\delta\psi_j(y)\delta\bar{K}_{AB}}\right) + \bar{\epsilon}\frac{\delta\Gamma}{\delta A_j}\delta(x, y)\right] = 0. \tag{23}
\end{aligned}$$

This is the non-trivial Ward-Takahashi identity for one point vertex function. Then we calculate the all the terms of the left hand side of Eq.(23) at one-loop level. We begin with the explicit computation of the first term in Eq.(23);

$$\begin{aligned}
\frac{\delta^2 W_1}{\delta\eta^{j'}(y')\delta\bar{K}_{\partial A}^\nu(x)} &= -\int d^4u \langle\bar{\psi}_{j'}(y')\sum_i\frac{1}{c_i}\partial_{\nu x}A_i(x)\cdot\psi_i(x)\mathcal{L}_{int}(u)\rangle \\
&= 2g\int d^4ue_u\langle\psi_{j'}(u)\bar{\psi}_{j'}(y')\rangle\sum_i\frac{1}{c_i}\langle\psi_i(x)\bar{\psi}_i(u)\rangle \\
&\quad \cdot\langle\partial_{\nu x}A_i(x)A_i(u)\rangle. \tag{24}
\end{aligned}$$



We obtain the first term in Eq.(23)

$$\begin{aligned}
\frac{\delta^2 \Gamma_1}{\delta \psi_j(y) \delta \bar{K}_{\partial A}^\nu(x)} &= -2ig \sum_i \frac{1}{c_i} \langle \psi_i(x) \bar{\psi}_i(y) \rangle \langle \partial_{\nu x} A_i(x) A_i(y) \rangle \\
&= 2ig \sum_i \frac{c_i}{i\mathcal{D}_x + m_i} \delta(x, y) \cdot \frac{\partial_{\nu x}}{\square_x + \xi \mathcal{R} - m_i^2} \delta(x, y). \quad (25)
\end{aligned}$$

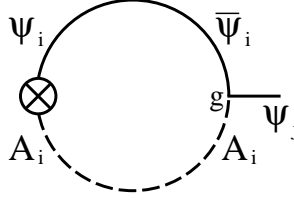


Figure 1: One Point Function  $\langle \psi \rangle_{tr}$  (i)

This diagram (Figure 1) is quadratically divergent. But it does not contribute to the effective action at all because we set all the source terms zero.

Also for the second term, via the connected two point function:

$$\begin{aligned}
\frac{\delta^2 W_1}{\delta \eta^{j'}(y') \delta \bar{K}_{\partial B}^\nu(x)} &= - \int d^4 u \langle \bar{\psi}_{j'}(y') \sum_i \frac{1}{c_i} \partial_{\nu x} B_i(x) \cdot \psi_i(x) \mathcal{L}_{int}(u) \rangle \\
&= -2g \int d^4 u e_u \langle \psi_{j'}(u) \bar{\psi}_{j'}(y') \rangle \sum_i \frac{1}{c_i} \langle \psi_i(x) \bar{\psi}_i(u) \rangle \\
&\quad \cdot \gamma_5 \langle \partial_{\nu x} B_i(x) B_i(u) \rangle, \quad (26)
\end{aligned}$$

and we obtain (Figure 2)

$$\begin{aligned}
\frac{\delta^2 \Gamma_1}{\delta \psi_j(y) \delta \bar{K}_{\partial B}^\nu(x)} &= 2ig \sum_i \frac{1}{c_i} \langle \psi_i(x) \bar{\psi}_i(y) \rangle \gamma_5 \langle \partial_{\nu x} B_i(x) B_i(y) \rangle \\
&= -2ig \sum_i \frac{c_i}{i\mathcal{D}_x + m_i} \delta(x, y) \gamma_5 \cdot \frac{\partial_{\nu x}}{\square_x + \xi \mathcal{R} - m_i^2} \delta(x, y). \quad (27)
\end{aligned}$$

Likewise we obtain

$$\begin{aligned}
\frac{\delta^2 \Gamma_1}{\delta \psi_j(y) \delta \bar{K}_A(x)} &= -2ig \sum_i \frac{m_i}{c_i} \langle \psi_i(x) \bar{\psi}_i(y) \rangle \langle A_i(x) A_i(y) \rangle \\
&= 2ig \sum_i \frac{c_i m_i}{i\mathcal{D}_x + m_i} \delta(x, y) \cdot \frac{1}{\square_x + \xi \mathcal{R} - m_i^2} \delta(x, y), \quad (28)
\end{aligned}$$

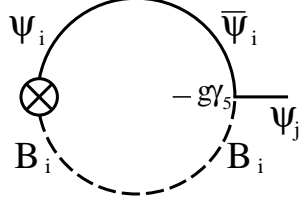


Figure 2: One Point Function  $\langle\psi\rangle_{tr}$  (ii)

$$\begin{aligned} \frac{\delta^2\Gamma_1}{\delta\psi_j(y)\delta\bar{K}_B(x)} &= 2ig \sum_i \frac{m_i}{c_i} \langle\psi_i(x)\bar{\psi}_i(y)\rangle \gamma_5 \langle B_i(x)B_i(y)\rangle \\ &= -2ig \sum_i \frac{c_i m_i}{i\mathcal{D}_x + m_i} \delta(x,y) \gamma_5 \cdot \frac{1}{\square_x + \xi\mathcal{R} - m_i^2} \delta(x,y), \end{aligned} \quad (29)$$

$$\begin{aligned} \frac{\delta^2\Gamma_1}{\delta\psi_j(y)\delta\bar{K}'_A(x)} &= -2ig \sum_i \frac{\xi}{c_i} \langle\psi_i(x)\bar{\psi}_i(y)\rangle \langle A_i(x)A_i(y)\rangle \\ &= 2ig \sum_i \frac{c_i \xi}{i\mathcal{D}_x + m_i} \delta(x,y) \cdot \frac{1}{\square_x + \xi\mathcal{R} - m_i^2} \delta(x,y), \end{aligned} \quad (30)$$

and

$$\begin{aligned} \frac{\delta^2\Gamma_1}{\delta\psi_j(y)\delta\bar{K}'_B(x)} &= 2ig \sum_i \frac{\xi}{c_i} \langle\psi_i(x)\bar{\psi}_i(y)\rangle \gamma_5 \langle B_i(x)B_i(y)\rangle \\ &= -2ig \sum_i \frac{c_i \xi}{i\mathcal{D}_x + m_i} \delta(x,y) \gamma_5 \cdot \frac{1}{\square_x + \xi\mathcal{R} - m_i^2} \delta(x,y). \end{aligned} \quad (31)$$

Finally we calculate the one point vertex function of the real scalar  $A$ . One point function is

$$\begin{aligned} \frac{\delta W_1}{\delta J_A^{j'}(y')} &= i \int d^4u \langle A_{j'}(y') \mathcal{L}_{int}(u) \rangle \\ &= ig \int d^4u e_u \sum_i \langle A_{j'}(y') A_{j'}(u) \rangle \left[ -\text{tr} \langle \psi_i(u) \bar{\psi}_i(u) \rangle \right. \\ &\quad \left. + 2 \langle F_i(u) A_i(u) \rangle - 2 \langle G_i(u) B_i(u) \rangle \right]. \end{aligned} \quad (32)$$

We obtain the non-vanishing one point vertex function (Figure 3)

$$\begin{aligned} \frac{\delta\Gamma_1}{\delta A_j(y)} &= g \sum_i \left[ \text{tr} \langle \psi_i(y) \bar{\psi}_i(y) \rangle - 2 \langle F_i(y) A_i(y) \rangle + 2 \langle G_i(y) B_i(y) \rangle \right] \\ &= ig \sum_i c_i \left[ \text{tr} \frac{1}{i\mathcal{D}_y + m_i} + \frac{4m_i}{\square_y + \xi\mathcal{R} - m_i^2} \right] \delta(y,y). \end{aligned} \quad (33)$$

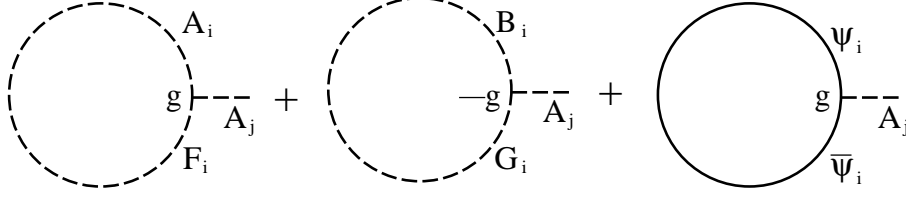


Figure 3: One Point Function  $\langle A \rangle_{tr}$

Inserting all of them into left hand side of the Ward-Takahashi identity for one point function Eq.(23) we can see that it is satisfied.<sup>7</sup> Note that the third line in Eq.(23) is zero itself.

## 2.4 Two Point Function

In this subsection we check the Ward-Takahashi identity for two point vertex function. We explicitly check one of them, Ward-Takahashi identity between real scalar and spinor two point functions because others are done in the same way. To do that we differentiate Eq.(22) with respect to  $\psi_j(y)$  and  $A_k(z)$  and set all the fields and the sources of composite fields zero. We obtain

$$\begin{aligned}
& \int d^4x e \left[ -\mathcal{D}_\mu \bar{\epsilon} \cdot \left\{ \gamma^\nu \gamma^\mu \left( \frac{\delta^3 \Gamma}{\delta A_k(z) \delta \psi_j(y) \delta \bar{K}_{\partial A}^\nu} + \gamma_5 \frac{\delta^3 \Gamma}{\delta A_k(z) \delta \psi_j(y) \delta \bar{K}_{\partial B}^\nu} \right) \right. \right. \\
& \quad \left. \left. - i \gamma^\mu \left( \frac{\delta^3 \Gamma}{\delta A_k(z) \delta \psi_j(y) \delta \bar{K}_A} - \gamma_5 \frac{\delta^3 \Gamma}{\delta A_k(z) \delta \psi_j(y) \delta \bar{K}_B} \right) \right\} \right. \\
& \quad \left. - \bar{\epsilon} \mathcal{R} \left( \frac{\delta^3 \Gamma}{\delta A_k(z) \delta \psi_j(y) \delta \bar{K}'_A} + \gamma_5 \frac{\delta^3 \Gamma}{\delta A_k(z) \delta \psi_j(y) \delta \bar{K}'_B} \right) + \mathcal{D}_\mu \bar{\epsilon} \cdot i \gamma^\mu \left( \right. \right. \\
& \quad \left. \left. \frac{\delta^3 \Gamma}{\delta A_k(z) \delta \psi_j(y) \delta \bar{K}_{A^2}} - \frac{\delta^3 \Gamma}{\delta A_k(z) \delta \psi_j(y) \delta \bar{K}_{B^2}} - \gamma_5 \frac{\delta^3 \Gamma}{\delta A_k(z) \delta \psi_j(y) \delta \bar{K}_{AB}} \right) \right. \\
& \quad \left. + \bar{\epsilon} \left\{ \frac{\delta^2 \Gamma}{\delta A_k(z) \delta A_j} \delta(x, y) + i \gamma^\mu \frac{\delta^2 \Gamma}{\delta \psi_j(y) \delta \bar{\psi}_k} \partial_\mu \delta(x, z) \right\} \right] = 0. \tag{34}
\end{aligned}$$

The first term is obtained from a two point function:

$$\frac{\delta^3 W_1}{\delta J_A^{k'}(z') \delta \eta^{j'}(y') \delta \bar{K}_{\partial A}^\nu(x)}$$

<sup>7</sup> The inclusion of linear  $A_i$  term in the potential amounts to a trivial relation like Eq.(19) at tree level

$$\begin{aligned}
&= \frac{1}{2} \int d^4u d^4v \sum_i \frac{1}{c_i} \langle A_{k_1}(z_1) \bar{\psi}_{j_1}(y_1) \partial_{\nu x} A_i(x) \cdot \psi_i(x) \mathcal{L}_{int}(u) \mathcal{L}_{int}(v) \rangle \\
&= -4g^2 \int d^4u d^4v e_u e_v \sum_{il} \frac{1}{c_i} \left[ \langle A_{k'}(z') A_{k'}(u) \rangle \langle \psi_{j'}(v) \bar{\psi}_{j'}(y') \rangle \right. \\
&\quad \cdot \langle \partial_{\nu x} A_i(x) A_i(v) \rangle \langle \psi_i(x) \bar{\psi}_i(u) \rangle \langle \psi_l(x) \bar{\psi}_l(v) \rangle + \langle A_{k'}(z') A_{k'}(v) \rangle \\
&\quad \cdot \langle \psi_{j'}(u) \bar{\psi}_{j'}(y') \rangle \left\{ \langle \partial_{\nu x} A_i(x) A_i(v) \rangle \langle A_l(u) F_l(v) \rangle + \langle \partial_{\nu x} A_i(x) F_i(v) \rangle \right. \\
&\quad \left. \left. \cdot \langle A_l(u) A_l(v) \rangle \right\} \langle \psi_i(x) \bar{\psi}_i(u) \rangle + \dots \right]. \tag{35}
\end{aligned}$$

The irrelevant part is omitted in the above expression. From the above equation we can read off the corresponding vertex function (Figure 4)

$$\begin{aligned}
&\frac{\delta^3 \Gamma_1}{\delta A_k(z) \delta \psi_j(y) \delta \bar{K}_{\partial A}^\nu(x)} \\
&= 4g^2 \sum_{il} \frac{1}{c_i} \left[ \langle \partial_{\nu x} A_i(x) A_i(y) \rangle \langle \psi_i(x) \bar{\psi}_i(z) \rangle \langle \psi_l(z) \bar{\psi}_l(y) \rangle + \left\{ \langle \partial_{\nu x} A_i(x) A_i(z) \rangle \right. \right. \\
&\quad \left. \left. \cdot \langle A_l(y) F_l(z) \rangle + \langle \partial_{\nu x} A_i(x) F_i(z) \rangle \langle A_l(y) A_l(z) \rangle \right\} \langle \psi_i(x) \bar{\psi}_i(y) \rangle \right] \\
&= -4ig^2 \sum_{il} c_i c_l \left[ \frac{\partial_{\nu x}}{\square_x + \xi \mathcal{R} - m_i^2} \delta(x, y) \cdot \frac{1}{i\mathcal{D}_x + m_i} \delta(x, z) \right. \\
&\quad \cdot \frac{1}{i\mathcal{D}_z + m_l} \delta(z, y) - \left\{ \frac{\partial_{\nu x}}{\square_x + \xi \mathcal{R} - m_i^2} \delta(x, z) \cdot \frac{m_l}{\square_y + \xi \mathcal{R} - m_l^2} \delta(y, z) \right. \\
&\quad \left. \left. + \frac{m_i \partial_{\nu x}}{\square_x + \xi \mathcal{R} - m_i^2} \delta(x, z) \cdot \frac{1}{\square_y + \xi \mathcal{R} - m_l^2} \delta(y, z) \right\} \frac{1}{i\mathcal{D}_x + m_i} \delta(x, y) \right], \tag{36}
\end{aligned}$$

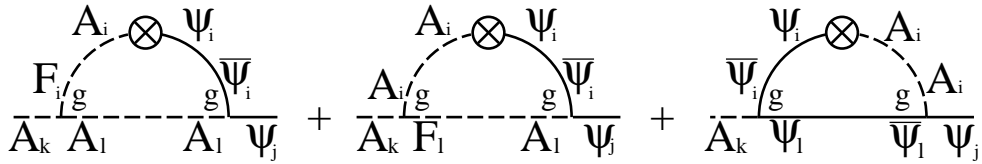


Figure 4: Two Point Function  $\langle \psi A \rangle_{tr(i)}$

which is linearly divergent. In the same way the two point vertex functions with composite operators are calculated (Figure 4 and 5):

$$\frac{\delta^3 \Gamma_1}{\delta A_k(z) \delta \psi_j(y) \delta \bar{K}_{\partial B}^\nu(x)}$$

$$\begin{aligned}
&= -4g^2 \sum_{il} \frac{1}{c_i} \left[ \langle \partial_{\nu x} B_i(x) B_i(y) \rangle \langle \psi_i(x) \bar{\psi}_i(z) \rangle \langle \psi_l(z) \bar{\psi}_l(y) \rangle - \left\{ \langle \partial_{\nu x} B_i(x) B_i(z) \rangle \right. \right. \\
&\quad \cdot \langle B_l(y) G_l(z) \rangle + \langle \partial_{\nu x} B_i(x) G_i(z) \rangle \langle B_l(y) B_l(z) \rangle \left. \right\} \langle \psi_i(x) \bar{\psi}_i(y) \rangle \Big] \gamma_5 \\
&= 4ig^2 \sum_{il} c_i c_l \left[ \frac{\partial_{\nu x}}{\square_x + \xi \mathcal{R} - m_i^2} \delta(x, y) \cdot \frac{1}{i\mathcal{D}_x + m_i} \delta(x, z) \right. \\
&\quad \cdot \frac{1}{i\mathcal{D}_z + m_l} \delta(z, y) - \left\{ \frac{\partial_{\nu x}}{\square_x + \xi \mathcal{R} - m_i^2} \delta(x, z) \cdot \frac{m_l}{\square_y + \xi \mathcal{R} - m_l^2} \delta(y, z) \right. \\
&\quad \left. \left. + \frac{m_i \partial_{\nu x}}{\square_x + \xi \mathcal{R} - m_i^2} \delta(x, z) \cdot \frac{1}{\square_y + \xi \mathcal{R} - m_l^2} \delta(y, z) \right\} \frac{1}{i\mathcal{D}_x + m_i} \delta(x, y) \right] \gamma_5, \quad (37)
\end{aligned}$$

Figure 5: Two Point Function  $\langle \psi A \rangle_{tr(ii)}$

$$\begin{aligned}
&\frac{\delta^3 \Gamma_1}{\delta A_k(z) \delta \psi_j(y) \delta \bar{K}_A(x)} \\
&= 4g^2 \sum_{il} \frac{m_i}{c_i} \left[ \langle A_i(x) A_i(y) \rangle \langle \psi_i(x) \bar{\psi}_i(z) \rangle \langle \psi_l(z) \bar{\psi}_l(y) \rangle + \left\{ \langle A_i(x) A_i(z) \rangle \right. \right. \\
&\quad \cdot \langle A_l(y) F_l(z) \rangle + \langle A_i(x) F_i(z) \rangle \langle A_l(y) A_l(z) \rangle \left. \right\} \langle \psi_i(x) \bar{\psi}_i(y) \rangle \Big] \\
&= -4ig^2 \sum_{il} c_i c_l m_i \left[ \frac{1}{i\mathcal{D}_x + m_i} \delta(x, z) \cdot \frac{1}{i\mathcal{D}_z + m_l} \delta(z, y) \frac{1}{\square_x + \xi \mathcal{R} - m_i^2} \delta(x, y) \right. \\
&\quad \left. - \frac{1}{\square_x + \xi \mathcal{R} - m_i^2} \delta(x, z) \cdot \frac{m_i + m_l}{\square_z + \xi \mathcal{R} - m_l^2} \delta(z, y) \frac{1}{i\mathcal{D}_x + m_i} \delta(x, y) \right], \quad (38)
\end{aligned}$$

$$\begin{aligned}
&\frac{\delta^3 \Gamma_1}{\delta A_k(z) \delta \psi_j(y) \delta \bar{K}_B(x)} \\
&= -4g^2 \sum_{il} \frac{m_i}{c_i} \left[ \langle B_i(x) B_i(y) \rangle \langle \psi_i(x) \bar{\psi}_i(z) \rangle \langle \psi_l(z) \bar{\psi}_l(y) \rangle - \left\{ \langle B_i(x) B_i(z) \rangle \right. \right. \\
&\quad \cdot \langle B_l(y) G_l(z) \rangle + \langle B_i(x) G_i(z) \rangle \langle B_l(y) B_l(z) \rangle \left. \right\} \langle \psi_i(x) \bar{\psi}_i(y) \rangle \Big] \gamma_5
\end{aligned}$$

$$\begin{aligned}
&= 4ig^2 \sum_{il} c_i c_l m_i \left[ \frac{1}{i\mathcal{D}_x + m_i} \delta(x, z) \cdot \frac{1}{i\mathcal{D}_z + m_l} \delta(z, y) \frac{1}{\square_x + \xi\mathcal{R} - m_i^2} \delta(x, y) \right. \\
&\quad \left. - \frac{1}{\square_x + \xi\mathcal{R} - m_i^2} \delta(x, z) \cdot \frac{m_i + m_l}{\square_z + \xi\mathcal{R} - m_l^2} \delta(z, y) \frac{1}{i\mathcal{D}_x + m_i} \delta(x, y) \right] \gamma^5, \quad (39)
\end{aligned}$$

$$\begin{aligned}
&\frac{\delta^3 \Gamma_1}{\delta A_k(z) \delta \psi_j(y) \delta \bar{K}'_A(x)} \\
&= 4g^2 \sum_{il} \frac{\xi}{c_i} \left[ \langle A_i(x) A_i(y) \rangle \langle \psi_i(x) \bar{\psi}_i(z) \rangle \langle \psi_l(z) \bar{\psi}_l(y) \rangle + \left\{ \langle A_i(x) A_i(z) \rangle \right. \right. \\
&\quad \left. \cdot \langle A_l(y) F_l(z) \rangle + \langle A_i(x) F_i(z) \rangle \langle A_l(y) A_l(z) \rangle \right\} \langle \psi_i(x) \bar{\psi}_i(y) \rangle \Big] \\
&= -4ig^2 \sum_{il} c_i c_l \xi \left[ \frac{1}{i\mathcal{D}_x + m_i} \delta(x, z) \cdot \frac{1}{i\mathcal{D}_z + m_l} \delta(z, y) \cdot \frac{1}{\square_x + \xi\mathcal{R} - m_i^2} \delta(x, y) \right. \\
&\quad \left. - \frac{1}{\square_x + \xi\mathcal{R} - m_i^2} \delta(x, z) \cdot \frac{m_i + m_l}{\square_z + \xi\mathcal{R} - m_l^2} \delta(z, y) \cdot \frac{1}{i\mathcal{D}_x + m_i} \delta(x, y) \right], \quad (40)
\end{aligned}$$

and

$$\begin{aligned}
&\frac{\delta^3 \Gamma_1}{\delta A_k(z) \delta \psi_j(y) \delta \bar{K}'_B(x)} \\
&= -4g^2 \sum_{il} \frac{\xi}{c_i} \left[ \langle B_i(x) B_i(y) \rangle \langle \psi_i(x) \bar{\psi}_i(z) \rangle \langle \psi_l(z) \bar{\psi}_l(y) \rangle - \left\{ \langle B_i(x) B_i(z) \rangle \right. \right. \\
&\quad \left. \cdot \langle B_l(y) G_l(z) \rangle + \langle B_i(x) G_i(z) \rangle \langle B_l(y) B_l(z) \rangle \right\} \langle \psi_i(x) \bar{\psi}_i(y) \rangle \Big] \gamma_5 \\
&= 4ig^2 \sum_{il} c_i c_l \xi \left[ \frac{1}{i\mathcal{D}_x + m_i} \delta(x, z) \cdot \frac{1}{i\mathcal{D}_z + m_l} \delta(z, y) \cdot \frac{1}{\square_x + \xi\mathcal{R} - m_i^2} \delta(x, y) \right. \\
&\quad \left. - \frac{1}{\square_x + \xi\mathcal{R} - m_i^2} \delta(x, z) \cdot \frac{m_i + m_l}{\square_z + \xi\mathcal{R} - m_l^2} \delta(z, y) \cdot \frac{1}{i\mathcal{D}_x + m_i} \delta(x, y) \right] \gamma^5. \quad (41)
\end{aligned}$$

A two point function with a composite operators( $\bar{K}_{A^2}$ ) is

$$\begin{aligned}
&\frac{\delta^3 W_1}{\delta J_A^{k'}(z') \delta \eta^{j'}(y') \delta \bar{K}_{A^2}(x)} \\
&= -ig \int d^4 u \sum_{ijk} \langle A_{k'}(z') \bar{\psi}_{j'}(y') A_i(x) A_j(x) \psi_k(x) \mathcal{L}_{int}(u) \rangle \\
&= 4ig^2 \int d^4 u e_u \sum_{jk} \left[ \langle A_{k'}(z') A_{k'}(x) \rangle \langle \psi_{j'}(u) \bar{\psi}_{j'}(y') \rangle \langle A_j(x) A_j(u) \rangle \right. \\
&\quad \cdot \langle \psi_k(x) \bar{\psi}_k(u) \rangle + \langle A_{k'}(z') A_{k'}(u) \rangle \langle \psi_{j'}(x) \bar{\psi}_{j'}(y') \rangle \langle A_j(x) A_j(u) \rangle \\
&\quad \left. \cdot \langle A_k(x) F_k(u) \rangle + \dots \right]
\end{aligned}$$

$$\begin{aligned}
&= 4ig^2 \int d^4u d^4v e_u e_v \sum_{jk} \left[ \left\{ \delta(x, v) \langle A_j(v) A_j(u) \rangle \langle \psi_k(v) \bar{\psi}_k(u) \rangle \right. \right. \\
&\quad \left. \left. + \delta(x, v) \langle A_j(v) A_j(u) \rangle \langle A_k(v) F_k(u) \rangle \right\} \langle A_{k'}(z') A_{k'}(v) \rangle \right. \\
&\quad \left. \cdot \langle \psi_{j'}(u) \bar{\psi}_{j'}(y') \rangle + \dots \right]. \tag{42}
\end{aligned}$$

We obtain (Figure 6)

$$\begin{aligned}
&\frac{\delta^3 \Gamma_1}{\delta A_k(z) \delta \psi_j(y) \delta \bar{K}_{A^2}(x)} \\
&= -4ig^2 \sum_{il} \langle A_l(z) A_l(y) \rangle \left[ \delta(x, z) \langle \psi_i(z) \bar{\psi}_i(y) \rangle + \delta(x, y) \langle A_i(y) F_i(z) \rangle \right] \\
&= 4ig^2 \sum_{il} c_i c_l \frac{1}{\square_x + x i\mathcal{R} - m_l^2} \delta(z, y) \left[ \delta(x, z) \frac{1}{i\mathcal{P}_z + m_i} \delta(z, y) \right. \\
&\quad \left. - \delta(x, y) \frac{m_i}{\square_y + \xi \mathcal{R} - m_i^2} \delta(y, z) \right]. \tag{43}
\end{aligned}$$

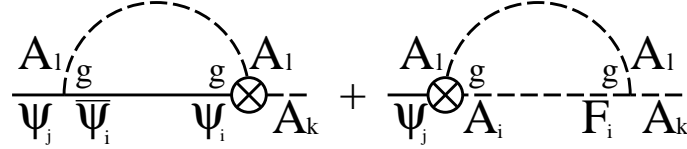


Figure 6: Two Point Function  $\langle \psi A \rangle_{tr}(\text{iii})$

In the same way we obtain (Figure 7 and 8)

$$\begin{aligned}
&\frac{\delta^3 \Gamma_1}{\delta A_k(z) \delta \psi_j(y) \delta \bar{K}_{B^2}(x)} = 4ig^2 \sum_{il} \delta(x, y) \langle B_i(y) B_i(z) \rangle \langle B_l(y) B_l(z) \rangle \\
&= -4ig^2 \sum_{il} c_i c_l \delta(x, y) \frac{m_i}{\square_y + \xi \mathcal{R} - m_i^2} \delta(y, z) \cdot \frac{1}{\square_z + \xi \mathcal{R} - m_l^2} \delta(z, y), \tag{44}
\end{aligned}$$

and

$$\begin{aligned}
&\frac{\delta^3 \Gamma_1}{\delta A_k(z) \delta \psi_j(y) \delta \bar{K}_{AB}(x)} = 4ig^2 \sum_{il} \delta(x, z) \langle B_i(z) B_i(y) \rangle \langle \psi_l(z) \bar{\psi}_l(y) \rangle \gamma_5 \\
&= -4ig^2 \sum_{il} c_i c_l \delta(x, z) \frac{1}{\square_z + \xi \mathcal{R} - m_i^2} \delta(z, y) \cdot \frac{1}{i\mathcal{P}_z + m_l} \delta(z, y) \gamma_5. \tag{45}
\end{aligned}$$

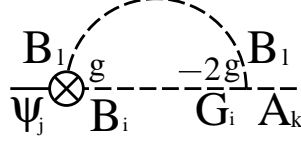


Figure 7: Two Point Function  $\langle \psi A \rangle_{tr(iv)}$

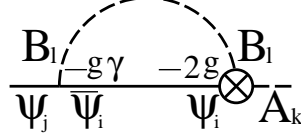


Figure 8: Two Point Function  $\langle \psi A \rangle_{tr(v)}$

Finally we compute the real scalar two point vertex function (Figure 9) and spinor one (Figure 10)

$$\begin{aligned} \frac{\delta^2 \Gamma_1}{\delta A_k(z) \delta A_j(x)} &= -2ig^2 \sum_{il} c_i c_l \left[ \text{tr} \frac{1}{i\mathcal{D}_x + m_i} \delta(x, z) \cdot \frac{1}{i\mathcal{D}_z + m_l} \delta(z, x) \right. \\ &\quad - 4 \left\{ 1 + \frac{m_i^2}{\square_x + \xi \mathcal{R} - m_i^2} \right\} \delta(x, z) \cdot \frac{1}{\square_z + \xi \mathcal{R} - m_l^2} \delta(z, x) \\ &\quad \left. - 4 \frac{m_i}{\square_x + \xi \mathcal{R} - m_i^2} \delta(x, z) \cdot \frac{m_l}{\square_z + \xi \mathcal{R} - m_l^2} \delta(z, x) \right], \end{aligned} \quad (46)$$

and

$$\begin{aligned} \frac{\delta^2 \Gamma_1}{\delta \psi_j(y) \delta \bar{\psi}_k(x)} &= 4ig^2 \sum_{il} c_i c_l \frac{1}{\square_x + \xi \mathcal{R} - m_i^2} \delta(x, y) \left[ \frac{1}{i\mathcal{D}_x + m_l} \delta(x, y) \right. \\ &\quad \left. + \gamma_5 \frac{1}{i\mathcal{D}_x + m_l} \delta(x, y) \gamma_5 \right]. \end{aligned} \quad (47)$$

Inserting all the vertex functions into left hand side of Eq.(34), we found the Ward-Takahashi identity for two-point functions satisfied. Other Ward-Takahashi identities for two-point functions which are obtained differentiating Eq. (22) with respect to  $B_k(z)$ ,  $F_k(z)$ , or  $G_k(z)$  instead of  $A_k(z)$  can be checked in the same way.

If we change the superpotential (1) as follows:

$$P = \frac{1}{2} \sum_{\alpha} m_{\alpha} \phi_{\alpha}^2 + \frac{\sqrt{2}}{3} \sum_{\alpha\beta\gamma} g_{\alpha\beta\gamma} \phi_{\alpha} \phi_{\beta} \phi_{\gamma}, \quad (48)$$



$$\begin{aligned}
& \frac{\text{F}_i \text{F}_i}{\text{A}_k \text{A}_l} \xrightarrow{g} \frac{\text{F}_i \text{F}_i}{\text{A}_l \text{A}_j} + \frac{\text{F}_i \text{A}_i}{\text{A}_k \text{A}_l} \xrightarrow{g} \frac{\text{F}_i \text{A}_i}{\text{F}_l \text{A}_j} + \frac{\text{G}_i \text{G}_i}{\text{A}_k \text{B}_l} \xrightarrow{-2g} \frac{\text{G}_i \text{G}_i}{\text{B}_l \text{A}_j} \\
& + \frac{\text{G}_i \text{B}_i}{\text{A}_k \text{B}_l} \xrightarrow{-2g} \frac{\text{G}_i \text{B}_i}{\text{G}_l \text{A}_j} + \frac{\bar{\Psi}_i \bar{\Psi}_i}{\text{A}_k \bar{\Psi}_l} \xrightarrow{g} \frac{\bar{\Psi}_i \bar{\Psi}_i}{\bar{\Psi}_l \text{A}_j}
\end{aligned}$$

Figure 9: Two Point Function  $\langle AA \rangle_{tr}$

$$\frac{\bar{\Psi}_l \bar{\Psi}_l}{\bar{\Psi}_j \text{A}_i} \xrightarrow{g} \frac{\bar{\Psi}_l \bar{\Psi}_l}{\text{A}_i \bar{\Psi}_k} + \frac{\bar{\Psi}_l \bar{\Psi}_l}{\bar{\Psi}_j \text{B}_i} \xrightarrow{-g\gamma_5} \frac{\bar{\Psi}_l \bar{\Psi}_l}{\text{B}_i \bar{\Psi}_k}$$

Figure 10: Two Point Function  $\langle \psi \psi \rangle_{tr}$

then we obtain a Wess-Zumino model with many chiral multiplets which have various masses and coupling constants. Greek letters indicate physical fields. It is easily found that SUSY is broken softly in these models as well as the Wess-Zumino model with a chiral multiplet. Inclusion of their regulators is straightforward:

$$P = \frac{1}{2} \sum_i \sum_\alpha m_{\alpha i} \phi_{\alpha i}^2 + \frac{\sqrt{2}}{3} \sum_{ijk} \sum_{\alpha\beta\gamma} g_{\alpha\beta\gamma} \phi_{\alpha i} \phi_{\beta j} \phi_{\gamma k}. \quad (49)$$

The subscriptions  $\phi_{\alpha 0}$ ,  $\phi_{\beta 0}$ ,  $\phi_{\gamma 0}$  denote the physical fields and  $\phi_{\alpha i}$ ,  $\phi_{\beta j}$ ,  $\phi_{\gamma k}$  where  $ijk \neq 0$ , denote their regulators, respectively.

If we set the coupling constant  $g_{122} = g_{121} = g_{221} = \frac{1}{2}g$  and others zero we obtain the same model with Wess-Zumino model with the flat direction which we shall study in Section 3.

Setting  $gc_i \rightarrow g_{ij}c_i$  in the one point vertex functions and  $g^2 c_i c_l \rightarrow g_{ikl} g_{ijl} c_i c_l$  in the two point vertex functions we obtained above, we found that Pauli-Villars regularization preserves softly broken SUSY in the Wess-Zumino model (Eq. (13))

with general coupling  $(g_{ijk})$ .<sup>8</sup> Because the coupling in Eq. (49) can be regarded as the special case of Eq. (13), Pauli-Villars regularization is suitable to the Wess-Zumino models (Eq. (48)) in de Sitter space.

### 3 Wess-Zumino Model with Flat Direction (I)

We compute a one-loop effective potential along the flat direction by using Pauli-Villars regularization. The simplest Wess-Zumino model that has a flat direction is given by a Lagrangian with regulators:

$$\begin{aligned}
e^{-1}\mathcal{L} = & \sum_i \left[ \frac{1}{2c_{1i}} \left\{ A_{1i} \square A_{1i} + B_{1i} \square B_{1i} + i\bar{\psi}_{1i} \not{D} \psi_{1i} + F_{1i}^2 + G_{1i}^2 \right. \right. \\
& + \xi_1 \mathcal{R}(A_{1i}^2 + B_{1i}^2) + m_{1i}(2A_{1i}F_{1i} - 2B_{1i}G_{1i} + \bar{\psi}_{1i}\psi_{1i}) \left. \right\} \\
& + \{1 \leftrightarrow 2\} \Big] + g \sum_{ijk} \left[ (A_{1i}A_{2j} - B_{1i}B_{2j})F_{2k} - (B_{1i}A_{2j} + A_{1i}B_{2j})G_{2k} \right. \\
& + \frac{1}{2} \left\{ (A_{2i}A_{2j} - B_{2i}B_{2j})F_{1k} - 2A_{2i}B_{2j}G_{1k} \right\} + \bar{\psi}_{1i}(A_{2j} - \gamma_5 B_{2j})\psi_{2k} \\
& \left. + \frac{1}{2} \bar{\psi}_{2i}(A_{1j} - \gamma_5 B_{1j})\psi_{2k} \right]. \tag{50}
\end{aligned}$$

Flat directions are

$$A_{1i}, B_{1i} \neq 0 \quad \text{and} \quad A_{2i} = B_{2i} = 0, \tag{51}$$

where

$$\begin{aligned}
F_{1i} &= -m_{1i}A_{1i} - \frac{g}{2} \sum_{jk} (A_{2j}A_{2k} - B_{2j}B_{2k}) = 0, \\
G_{1i} &= m_{1i}B_{1i} + g \sum_{jk} A_{2j}B_{2k} = 0, \\
F_{2i} &= -m_{2i}A_{2i} - g \sum_{jk} (A_{1j}A_{2k} - B_{1j}B_{2k}) = 0, \\
G_{2i} &= m_{2i}B_{2i} + g \sum_{jk} (A_{1j}B_{2k} + A_{1k}B_{2j}) = 0, \tag{52}
\end{aligned}$$

if  $ms$  and  $\xi$ s are all equal zero. The Pauli-Villars constraints are  $\sum_i c_{1i}m_{1i}^p = 0$ , and  $\sum_i c_{2i}m_{2i}^p = 0$  for  $p = 0, 1, 2$ .

---

<sup>8</sup>In this case Pauli-Villars regularization does not regularize all the divergent diagrams.

We compute one and two point functions by using the regulators and n-point functions ( $n \geq 3$ ) without regulators, since one and two point functions are divergent while others are convergent in a effective potential. Using the same procedure of previous section we obtain the non-vanishing one point function:

$$\frac{\delta\Gamma_1}{\delta A_{1j}(y)} = \frac{g}{2} \sum_i \left[ \text{tr} \langle \psi_{2i}(y) \bar{\psi}_{2i}(y) \rangle - 4 \langle A_{2i}(y) F_{2i}(y) \rangle \right]. \quad (53)$$

We invoke the curvature expansion of propagators. The curvature expansion is sufficient for our calculations since the divergent parts are linear in curvature. Using propagators represented in momentum space in Eq. (8) and (10) we obtain

$$\frac{\delta\Gamma_1}{\delta A_{1j}(y)} = 2ig \sum_i c_{2i} m_{2i} \int \frac{d^4 k}{(2\pi)^4} \left( \frac{1}{4} - \xi_2 \right) \frac{\mathcal{R}}{(k^2 + m_{2i}^2)^2} + \mathcal{O}(\mathcal{R}^2) \quad (54)$$

$$= \frac{2g}{16\pi^2} \sum_i c_{2i} m_{2i} \left( \frac{1}{4} - \xi_2 \right) \mathcal{R} \ln \frac{m_{2i}^2}{\mu^2} + \mathcal{O}(\mathcal{R}^2), \quad (55)$$

where parameter  $\mu$  is just introduced to make the argument of logarithm dimensionless while it does not contribute to the result at all because of the Pauli-Villars constraint.

Next we calculate a two point function:

$$\begin{aligned} \frac{\delta^2\Gamma_1}{\delta A_{1k}(z) \delta A_{1j}(y)} &= -\frac{i}{2} g^2 \sum_{il} \left[ 4 \langle A_{2i}(z) A_{2i}(y) \rangle \langle F_{2l}(z) F_{2l}(y) \rangle \right. \\ &\quad + 4 \langle A_{2i}(z) F_{2i}(y) \rangle \langle F_{2l}(z) A_{2l}(y) \rangle \\ &\quad \left. - \text{tr} \langle \psi_{2i}(z) \bar{\psi}_{2i}(y) \rangle \langle \psi_{2l}(y) \bar{\psi}_{2l}(z) \rangle \right]. \end{aligned} \quad (56)$$

Using Riemann normal coordinates around  $x$ .

$$\begin{aligned} \frac{\delta^2\Gamma_1}{\delta A_{1k}(z) \delta A_{1i}(x)} &= 2ig^2 \sum_{jl} c_{2j} c_{2l} \int \frac{d^4 k}{(2\pi)^4} \frac{d^4 p}{(2\pi)^4} e^{i(k-p) \cdot (z-x)} \left[ -\frac{1}{k^2 + m_{2j}^2} \right. \\ &\quad + \frac{2(k \cdot p) + m_{2j}^2 + m_{2l}^2}{2(k^2 + m_{2j}^2)(p^2 + m_{2l}^2)} + \frac{(1 - 3\xi_2)\mathcal{R}}{3(k^2 + m_{2j}^2)^2} - \frac{2\mathcal{R}_{ab} k^a k^b}{3(k^2 + m_{2j}^2)^3} \\ &\quad + \mathcal{R} \left\{ -\frac{4(1 - 3\xi_2)(m_{2j} + m_{2l})^2 + 5(k \cdot p) - 2m_{2j}m_{2k}}{12(k^2 + m_{2j}^2)(p^2 + m_{2l}^2)^2} \right. \\ &\quad \left. \left. + \frac{p^2}{6} \frac{2(k \cdot p) + m_{2j}^2 + m_{2l}^2}{(k^2 + m_{2j}^2)(p^2 + m_{2l}^2)^3} \right\} + \mathcal{O}(\mathcal{R}^2) \right]. \end{aligned} \quad (57)$$

Setting  $k - p = q$  and performing volume integration of  $z$  we obtain

$$\int d^4 z e_z \frac{\delta^2\Gamma_1}{\delta A_{1k}(z) \delta A_{1i}(x)}$$

$$\begin{aligned}
&= 2ig^2 \sum_{jl} c_{2j} c_{2l} \int \frac{d^4 k}{(2\pi)^4} \mathcal{R} \left[ \frac{(1-4\xi_2)}{4(k^2 + m_{2j}^2)^2} + \frac{m_{2j}^2}{6(k^2 + m_{2j}^2)^3} \right. \\
&\quad \left. - \frac{6(1-4\xi_2)m_{2j}m_{2l} + 3(1-4\xi_2)m_{2j}^2 + 2(1-6\xi_2)m_{2l}^2}{12(k^2 + m_{2j}^2)(k^2 + m_{2l}^2)^2} \right] + \mathcal{O}(\mathcal{R}^2) \\
&= \frac{2g^2}{16\pi^2} \left( \frac{1}{4} - \xi_2 \right) \mathcal{R} \sum_{jl} c_{2j} c_{2l} \left[ \ln \frac{m_{2j}m_{2l}}{\mu^2} + \frac{m_{2j} + m_{2l}}{m_{2j} - m_{2l}} \ln \frac{m_{2j}}{m_{2l}} \right] + \mathcal{O}(\mathcal{R}^2). \quad (58)
\end{aligned}$$

At the final step we used the Pauli-Villars constraint  $\sum_j c_{2j} = 0$ . An effective action can be expanded in a field

$$\begin{aligned}
\Gamma[\phi(x)] &= \Gamma[0] + \int d^4 x e_x \frac{\delta\Gamma}{\delta\phi(x)} \Big|_{\phi=0} \phi(x) \\
&\quad + \frac{1}{2!} \int d^4 x d^4 y e_x e_y \frac{\delta^2\Gamma}{\delta\phi(x)\delta\phi(y)} \Big|_{\phi=0} \phi(x)\phi(y) + \dots \quad (59)
\end{aligned}$$

and for a constant field we obtain an effective potential:

$$-V(\phi) = -V(0) + \frac{\delta\Gamma}{\delta\phi(x)} \Big|_{\phi=0} \phi + \frac{1}{2!} \int d^4 y e_y \frac{\delta^2\Gamma}{\delta\phi(x)\delta\phi(y)} \Big|_{\phi=0} \phi^2 + \dots \quad (60)$$

We set  $m_{1i} = 0$  to obtain exact flat directions in flat space limit. We take  $\langle A_{10} \rangle = \phi$  and others are zero, since the direction is a flat direction at tree level if  $\xi_1$  vanishes. The effective potential along the direction is written as follows

$$\begin{aligned}
V_{eff}(\phi) &= -\frac{1}{\sqrt{2}} J\mathcal{R}\phi - \frac{1}{2}\xi_1 \mathcal{R}\phi^2 + \frac{2g}{16\pi^2} \sum_i c_{2i} m_{2i} \left( \frac{1}{4} - \xi_2 \right) \mathcal{R} \ln \frac{m_{2i}^2}{\mu^2} \cdot \phi \\
&\quad + \frac{g^2}{16\pi^2} \sum_{ij} c_{2i} c_{2j} \left( \frac{1}{4} - \xi_2 \right) \mathcal{R} \left[ \ln \frac{m_{2i}m_{2j}}{\mu^2} + \frac{m_{2i} + m_{2j}}{m_{2i} - m_{2j}} \ln \frac{m_{2i}}{m_{2j}} \right] \phi^2 \\
&\quad + \left[ f(\phi) - f'(0)\phi - \frac{1}{2!} f''(0)\phi^2 \right] \mathcal{R} + \mathcal{O}(\mathcal{R}^2), \quad (61)
\end{aligned}$$

where the term  $-J\mathcal{R}\phi/\sqrt{2}$  is introduced for renormalization of linear term. The linear term does not affect one-loop calculation. The last line in Eq. (61) corresponds to the interaction part ( $n \geq 3$ ) which are all convergent. The function  $f(\phi)$  is derived in the next section. We renormalize one and two point vertex function at flat space limit as follows:

$$\frac{\partial V_{eff}}{\partial\phi} \Big|_{\phi=M} = -\frac{1}{\sqrt{2}} J\mathcal{R} - \xi_1 \mathcal{R}M + \frac{2g}{16\pi^2} \sum_i c_{2i} m_{2i} \left( \frac{1}{4} - \xi_2 \right) \mathcal{R} \ln \frac{m_{2i}^2}{\mu^2}$$

$$\begin{aligned}
& + \frac{2g^2}{16\pi^2} \sum_{ij} c_{2i} c_{2j} \left( \frac{1}{4} - \xi_2 \right) \mathcal{R} \left[ \ln \frac{m_{2i} m_{2j}}{\mu^2} + \frac{m_{2i} + m_{2j}}{m_{2i} - m_{2j}} \ln \frac{m_{2i}}{m_{2j}} \right] M \\
& + \left[ f'(M) - f'(0) - f''(0)M \right] \mathcal{R}
\end{aligned} \tag{62}$$

$$= -\frac{1}{\sqrt{2}} J_R \mathcal{R} - \xi_{1R} \mathcal{R} M, \tag{63}$$

$$\begin{aligned}
\left. \frac{\partial^2 V_{eff}}{\partial \phi^2} \right|_{\phi=M} &= -\xi_1 \mathcal{R} + \frac{2g^2}{16\pi^2} \sum_{ij} c_{2i} c_{2j} \left( \frac{1}{4} - \xi_2 \right) \mathcal{R} \left[ \ln \frac{m_{2i} m_{2j}}{\mu^2} + \frac{m_{2i} + m_{2j}}{m_{2i} - m_{2j}} \ln \frac{m_{2i}}{m_{2j}} \right] \\
&+ \left[ f''(M) - f''(0) \right] \mathcal{R}
\end{aligned} \tag{64}$$

$$= -\xi_{1R} \mathcal{R}. \tag{65}$$

The effective potential is written in terms of renormalized parameters as follows

$$V_{eff}(\phi) = -\frac{1}{\sqrt{2}} J_R \mathcal{R} \phi - \frac{1}{2} \xi_{1R} \mathcal{R} \phi^2 + V_{one-loop}(\phi) + V_{counter}(\phi), \tag{66}$$

where

$$V_{counter}(\phi) \equiv \frac{1}{\sqrt{2}} (-J + J_R) \mathcal{R} \phi - \frac{1}{2} (\xi_1 - \xi_{1R}) \mathcal{R} \phi^2. \tag{67}$$

Finally we obtain the effective potential written in terms of renormalized parameters defined by Eq.(63) and (65) as follows

$$\begin{aligned}
V_{eff}(\phi) &= \left[ -\frac{1}{\sqrt{2}} J \phi - \frac{1}{2} \xi_1 \phi^2 + f(\phi) - f'(M)(\phi - M) - \frac{1}{2} f''(M)(\phi - M)^2 \right] \mathcal{R} \\
&+ \mathcal{O}(\mathcal{R}^2),
\end{aligned} \tag{68}$$

where we have omitted the subscript (R).

## 4 Wess-Zumino Model with Flat Direction (II)

In this section we calculate the effective potential by proper-time cutoff regularization. This regularization is exact in curvature, so it is applicable to the very early universe. Eliminating the auxiliary fields in the Lagrangian (Eq. (50)) without regulators, and setting  $m_1 = 0$  we obtain the corresponding Euclidean potential: <sup>9</sup>

$$V = \frac{1}{2} \xi_1 \mathcal{R} (A_1^2 + B_1^2) + \frac{1}{2} (\xi_2 \mathcal{R} - m_2^2) (A_2^2 + B_2^2) - g m_2 A_1 (A_2^2 + B_2^2)$$

---

<sup>9</sup>The Euclidean coordinate  $x^4 = ix^0$ ; the Euclidean gamma matrices  $\gamma_{E4} = \gamma_0$ ,  $\gamma_{Ei} = i\gamma_i$ , and  $\gamma_E^5 = -i\gamma_{E1}\gamma_{E2}\gamma_{E3}\gamma_{E4}$ . In our convention the form of bosonic part of Lagrangian is same as the Lorentzian one, so is potential. We shall omit the subscript  $E$  in the following.

	$L$	$m^2 a^2$
$A_1, B_1$	0	$12\xi_1$
$A_2, B_2$	0	$12\xi_2 + (m_2 + g\phi)^2 a^2$
$\psi_1$	1/2	0
$\psi_2$	1/2	$(m_2 + g\phi)^2 a^2$

Table 2: The mass spectrum of the fluctuations with the presence of the vacuum expectation value  $\phi$ .

$$-\frac{g^2}{8}(A_2^2 + B_2^2)^2 - \frac{g^2}{2}(A_1^2 + B_1^2)(A_2^2 + B_2^2). \quad (69)$$

In Euclidean de Sitter space ( $S^4$ )  $\mathcal{R} = -12a^{-2}$ , where  $a = H^{-1}$ . We decompose the scalar fields around the flat direction where  $\langle A_1 \rangle = \phi$  as

$$\phi_1 = \frac{\phi + A_1 + iB_1}{\sqrt{2}}, \quad \phi_2 = \frac{A_2 + iB_2}{\sqrt{2}}. \quad (70)$$

We obtain the Lagrangian of the fluctuation of quadratic order:

$$\begin{aligned} e^{-1}\mathcal{L}_2 = & -\frac{1}{2}\left[A_1(\square + \xi_1\mathcal{R})A_1 + B_1(\square + \xi_1\mathcal{R})B_1 - \bar{\psi}_1\not{D}\psi_1\right. \\ & + A_2\{\square + \xi_2\mathcal{R} - (m_2 + g\phi)^2\}A_2 + B_2\{\square + \xi_2\mathcal{R} - (m_2 + g\phi)^2\}B_2 \\ & \left. - \bar{\psi}_2\{\not{D} - (m_2 + g\phi)\}\psi_2\right], \end{aligned} \quad (71)$$

and we can read off the effective mass of the fluctuations that depends on the expectation value  $\phi$ . The result is summarized in Table 2. From the mass generating pattern in Table 2 and from Appendix C we can readily obtain the effective potential. Because the finite part of the effective potential,  $\zeta'(0)$ , cannot be expressed by elementary functions, we must invoke numerical integrations. The analytic evaluation is possible at flat space limit<sup>10</sup> From the general rule in Table 5 in Appendix C, we have

$$\begin{aligned} V_{eff}(\phi) = & \left[-\frac{1}{\sqrt{2}}Js\phi - \frac{1}{2}\xi_1\phi^2 + \frac{1-4\xi_2}{64\pi^2}(g\phi + m_2)^2\left\{\gamma - 1 - \ln\frac{\Lambda^2}{(g\phi + m_2)^2}\right\}\right]\mathcal{R} \\ & + \mathcal{O}(\mathcal{R}^2), \end{aligned} \quad (72)$$

<sup>10</sup>More precisely, at the limit,  $g^2\phi^2 a^2 \rightarrow \infty$  and  $m_2^2 a^2 \rightarrow \infty$ .

where we recover  $\mathcal{R}$  instead of  $-12a^{-2}$ . We can see that one and two point vertex functions are all the divergent parts. We adopt the same renormalization condition as Eq. (63) and (65). Finally we obtain the explicit form of effective potential

$$V_{eff}(\phi) = -\frac{1}{\sqrt{2}}J\mathcal{R}\phi - \frac{1}{2}\xi_1\mathcal{R}\phi^2 + \frac{1}{16\pi^2}\left(\frac{1}{4} - \xi_2\right)\mathcal{R}\left[(g\phi + m_2)^2 \cdot \left\{\ln\left(\frac{g\phi + m_2}{gM + m_2}\right)^2 - 1\right\} - 2(g\phi - gM)^2\right] + \mathcal{O}(\mathcal{R}^2) \quad (73)$$

up to constant. We omitted the subscript (R) in the above expression. Since terms of  $\phi^n$  ( $n \geq 3$ ) are independent of regularizations, we set

$$f(\phi) = -\frac{1 - 4\xi_2}{64\pi^2}(g\phi + m_2)^2 \ln \frac{\Lambda^2}{(g\phi + m_2)^2} \quad (74)$$

in Eq. (68). The right hand side of Eq. (74) is robbed from Eq. (72). We then find that the renormalized effective potential (68) is the same as one in Eq. (73) up to  $\mathcal{O}(\mathcal{R}^2)$  terms. This fact means that they give the same effective potential at all orders of the curvature, since difference between the two regularizations is to linear order in  $\mathcal{R}$ .<sup>11</sup> Quadratic and higher order terms in  $\mathcal{R}$  are convergent.

Note that using proper-time cutoff regularization, the effective action of fermions can not be generated. By means of it we can regularize only the one-loop contribution along which a kind of field runs while the effective action of fermions are made from the one-loop of several kinds of field. An example with a scalar self-coupling and the Yukawa coupling is depicted in Fig. (11). Thus we cannot check the Ward-Takahashi identity for softly broken SUSY by using proper-time cutoff regularization.

Since  $\xi_\alpha$  represent the strength of a coupling with (classical) gravity, it is plausible to assume  $\xi_1 = \xi_2 = \xi$ . For the flat direction, which has  $J \simeq 0$  and  $\xi \simeq 0$ , the effective potential becomes

$$V_{eff}(\phi) \sim \frac{\mathcal{R}}{64\pi^2}\left[(g\phi + m_2)^2\left\{\ln\left(\frac{g\phi + m_2}{gM + m_2}\right)^2 - 1\right\} - 2(g\phi - gM)^2\right] + \mathcal{O}(\mathcal{R}^2). \quad (75)$$

In the large  $\phi$  region the asymptotic form of the effective potential is

$$V_{eff}(\phi) \rightarrow \frac{g^2}{64\pi^2}\mathcal{R}\phi^2 \ln\left(\frac{\phi}{M}\right)^2, \quad (76)$$

---

<sup>11</sup>From the above argument we deduced that we can naively obtain the same effective potential (73) by means of any possible regularization. Accordingly, our effective potential can possess merits of all possible regularizations.

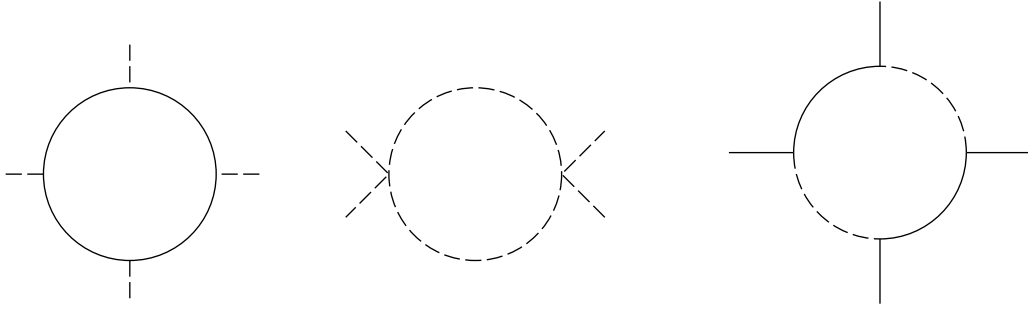


Figure 11: The left two is the contributions to scalar four point vertex function where the solid line denotes a spinor field and the dashed line denotes a scalar field. The remaining is the contribution to spinor four point function.

where  $\mathcal{R} = -12H^2$ . The effective potential becomes unbounded from below in the large  $\phi$  region. This behavior is true also in the large curvature region (Figure (12)). This is because there is  $\xi$  independent term in the effective potential. Since one-loop approximation is reliable so long as

$$\frac{g^2}{64\pi^2} \ln \left( \frac{\phi}{M} \right)^2 \lesssim \mathcal{O}(1), \quad (77)$$

the effective curvature coupling constant could effectively be order unity. Accordingly, the flat direction is no longer flat. If  $J$  or  $\xi$  is not so small, tree level potential is dominant. In this case no flat direction exists even at tree level (Figure (13)).

## 5 Conclusions and Discussion

We have found that SUSY in curved space is broken softly in the Wess-Zumino models. We consider that any suitable regularization must satisfy the Ward-Takahashi identity for softly broken SUSY. We found that Pauli-Villars regularization is a suitable regularization.

Using the regularization we calculated the one-loop effective potential along a flat direction in de Sitter space. We calculated it to linear order in the (space-time) curvature, or quadratic in the Hubble parameter, and found that it is unbounded from below as follows:

$$V_{eff}(\phi) \rightarrow -\frac{3g^2}{16\pi^2} H^2 \phi^2 \ln \left( \frac{\phi^2}{M^2} \right). \quad (78)$$



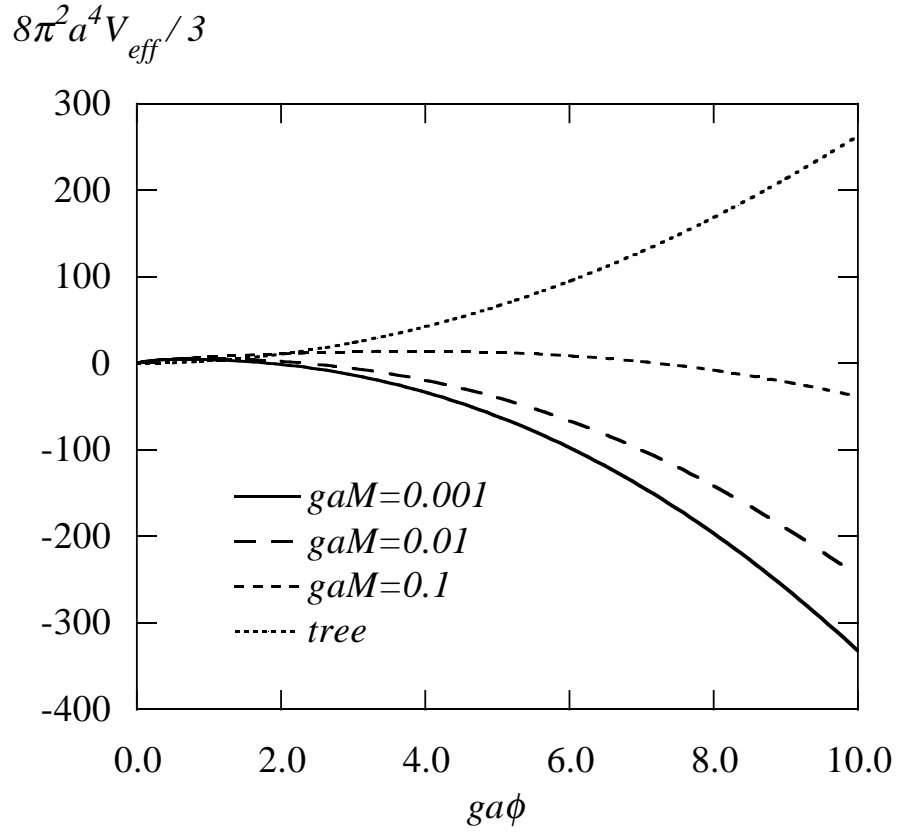


Figure 12: The one-loop effective potential of the Wess-Zumino model with  $g = 0.1$ ,  $J = 0$ ,  $12\xi_i = 0.001$ . We have changed only the renormalization point as  $gaM = 0.001, 0.01, 0.1$  with all the other parameters fixed. The tree potential is also depicted for a comparison. We fixed  $V_{eff}(0) = 0$ .

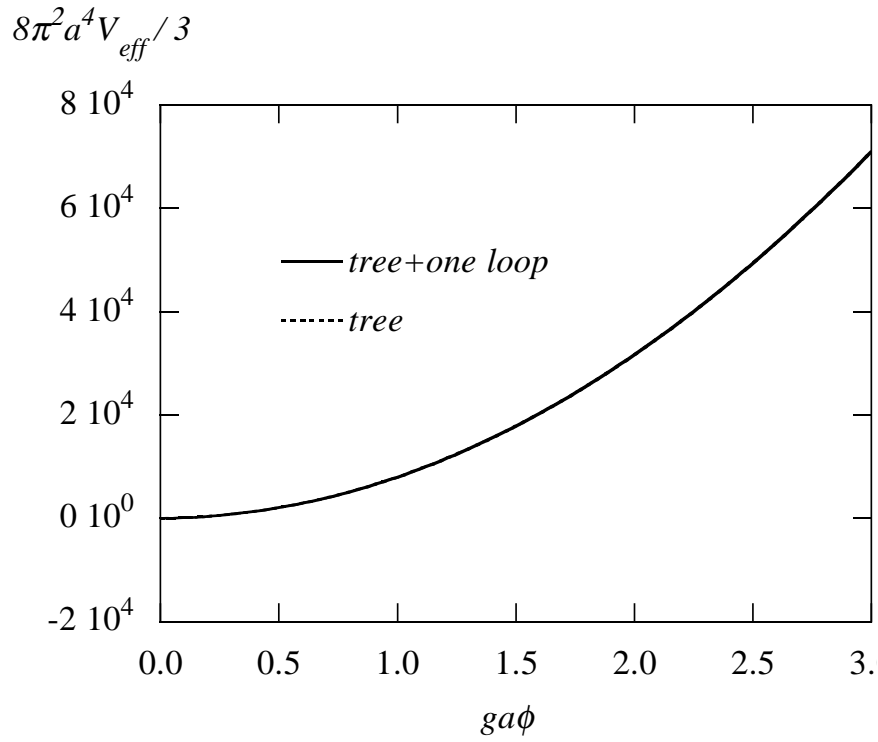


Figure 13: The one-loop effective potential of the Wess-Zumino model with  $g = 0.1$ ,  $J = 0$ ,  $\xi_i = 1/4$ . The tree potential is also depicted for a comparison.

This result means that the effective potential is not flat at large  $\phi$  region.

The effective potential is exactly same as that is obtained by using proper-time cutoff regularization which is exact in curvature. The agreement of the two effective potentials to linear order in the curvature is sufficient, since divergence and difference of regularizations appear only to the order. From the above argument the effective potential we obtained possesses softly broken SUSY and reliability in the large curvature region.

The numerical calculation is depicted in Figure (12). It behaves like Eq.(78) at the large  $\phi$  region. The outstanding shape of the effective potential disappears as  $H \rightarrow 0$ . If the flat direction is not exist at tree level, the potential does not receive much quantum correction (Figure (13)).

The form of the effective potential is reliable even after inclusion of the renormalization group effect[15]. We argued that asymptotic behavior of the one-loop effective potential is unavoidable irrespective of matter contents and detail of the model if the flat direction couples to a scalar multiplet[15].<sup>12</sup>

Let us apply the unbounded potential to the Affleck-Dine mechanism. The scalar field begins to roll down along the unbounded potential in the inflationary era[16]. After inflation and by the time when the Hubble parameter becomes comparable to the mass of the scalar field, the potential becomes bounded from below. The scalar field stops rolling and has a large value along the direction. After that it rolls down to a true minimum (the origin). Accordingly, our effective potential will favor their mechanism though it is not always flat.<sup>13</sup>

## 6 Acknowledgement

The author is grateful to H. Murayama, H. Suzuki, T. Yanagida, and J. Yokoyama. He also thanks K. Hikasa, M. Hotta, H. Inoue, K. Tobe, S. Watamura, M. Yamaguchi, and M. Yoshimura for useful comments.

---

<sup>12</sup>Although the effective potential seems to be unbounded from below, (super)gravity shall bound the potential at the larger  $\phi$  region.[15]

<sup>13</sup>In the future work we will discuss the application to the Affleck-Dine mechanism in detail.

## A Notations

In this paper we use the convention as follows:

$$\eta_{\mu\nu} = \text{diag}(- + + +) \quad (79)$$

$$e \equiv \sqrt{-g} = \sqrt{-|g_{\mu\nu}|} \quad (80)$$

$$\mathcal{R}^\rho_{\sigma\nu\mu} \equiv \partial_\mu \Gamma^\rho_{\nu\sigma} - \partial_\nu \Gamma^\rho_{\mu\sigma} + \Gamma^\lambda_{\nu\sigma} \Gamma^\rho_{\mu\lambda} - \Gamma^\lambda_{\mu\sigma} \Gamma^\rho_{\nu\lambda} = \mathcal{R}_{\mu\nu\sigma}{}^\rho \quad (81)$$

$$\mathcal{R}_{\nu\mu} \equiv \mathcal{R}^\rho_{\nu\rho\mu} \quad (82)$$

$$\mathcal{R} \equiv \mathcal{R}^\nu_{\nu}. \quad (83)$$

For the conventions of Gamma matrices are the same as those in Ref. [2]

## B Conformally transformed Wess-Zumino model

We investigate another reason why SUSY is broken softly in de Sitter space. De Sitter space can be written by a conformally flat metric  $g_{\mu\nu}(x) = \{\Omega(x)\}^2 \eta_{\mu\nu}$ .<sup>14</sup> Defining the tilde fields as

$$\begin{aligned} \tilde{A} &= \Omega(x)A, & \tilde{B} &= \Omega(x)B \\ \tilde{F} &= \{\Omega(x)\}^2 F, & \tilde{G} &= \{\Omega(x)\}^2 G \\ \tilde{\psi} &= \{\Omega(x)\}^{3/2} \psi, \end{aligned} \quad (84)$$

we obtain

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2}(\partial_\mu \tilde{A} \cdot \partial^\mu \tilde{A} + \partial_\mu \tilde{B} \cdot \partial^\mu \tilde{B}) + \frac{i}{2} \tilde{\bar{\psi}} \not{\partial} \tilde{\psi} + \frac{1}{2}(\tilde{F}^2 + \tilde{G}^2) \\ & + a^{-2} \Omega^2 (1 - 6\xi)(\tilde{A}^2 + \tilde{B}^2) + m\Omega(\tilde{A}\tilde{F} - \tilde{B}\tilde{G} + \frac{1}{2}\tilde{\bar{\psi}}\tilde{\psi}) \\ & + g[\tilde{\bar{\psi}}(\tilde{A} - \gamma_5 \tilde{B})\tilde{\psi} + \tilde{F}\tilde{A}^2 - \tilde{F}\tilde{B}^2 - 2\tilde{G}\tilde{A}\tilde{B}], \end{aligned} \quad (85)$$

$a$  is the radius of de Sitter space. This Lagrangian is corresponding to the Wess-Zumino model with a *soft SUSY breaking mass term* which depends on coordinates.

---

<sup>14</sup>It is known that the conformal flat metric do not cover the whole de Sitter manifold. So we need to attach the patches appropriately. But we ignore this subtlety here.

	$6 \times \text{Res}[\zeta(2)]$	$6 \times \text{Res}[\zeta(1)]$	$180 \times \zeta(0)$
real scalar	1	$-m^2 a^2 + 2$	$15m^4 a^4 - 60m^2 a^2 + 58$
Majorana spinor	2	$-2m^2 a^2 - 2$	$30m^4 a^4 + 60m^2 a^2 + 11$
transverse massive vector	3	$-3m^2 a^2 + 3$	$45m^4 a^4 - 90m^2 a^2 - 21$
Rarita-Schwinger	4	$-4m^2 a^2 - 16$	$60m^4 a^4 + 480m^2 a^2 + 802$
symmetric transverse	5	$-5m^2 a^2 - 10$	$75m^4 a^4 + 300m^2 a^2 - 10$
traceless tensor			

Table 3: Some values of the generalized zeta function

## C Proper-time cutoff

A one-loop effective potential is given by [15, 17]

$$\begin{aligned}
V_{eff}(\phi) = & V(\phi) + \frac{3}{16\pi^2} \left\{ -\frac{1}{2} \text{Res}[\zeta(2)] \Lambda^4 - \text{Res}[\zeta(1)] a^{-2} \Lambda^2 \right. \\
& \left. + \zeta(0) [\gamma - \ln(a\Lambda)^2] a^{-4} - \zeta'(0) a^{-4} \right\}.
\end{aligned} \tag{86}$$

$V(\phi)$  is a tree-level potential and  $\zeta(n)$  is generalized zeta function [18] on  $S^4$  evaluated by Allen [19]. It is convenient to translate his results in terms of effective masses in the wave operator  $\Delta = -\square + m^2$ . For spinorial fields  $\Delta = (\mathcal{D} - m)^\dagger (\mathcal{D} - m)$ . Some relevant values are summarized in Table 3. On the other hand, the derivative of zeta function at  $s = 0$ ,  $\zeta'(0)$ , is given by [19]

$$\begin{aligned}
\zeta'(0) = & -\frac{1}{3}(2L+1) \int_0^{y(L)} \left[ x^3 + \left(L + \frac{1}{2}\right)^2 x \right] \left[ \psi\left(L + \frac{1}{2} + ix\right) \right. \\
& \left. + \psi\left(L + \frac{1}{2} - ix\right) \right] dx + c(L) + \frac{2}{3}(2L+1) \left[ \zeta'_R\left(-3, L + \frac{3}{2}\right) \right. \\
& \left. - \left(L + \frac{1}{2}\right)^2 \zeta'_R\left(-1, L + \frac{3}{2}\right) \right],
\end{aligned} \tag{87}$$

where constants  $y(L)$  and  $c(L)$  are given in Table 4. In the above expression,  $\psi(s)$  is the digamma function and  $\zeta_R(s, \alpha)$  is the extended Riemann's zeta function. The integral in Eq.(87) cannot be done analytically but we can evaluate the asymptotic behavior at  $m^2 a^2 \rightarrow \infty$  by using the asymptotic form of the digamma function [20]. We may also evaluate the integral Eq.(87) numerically. In Table 5, the asymptotic form of  $\zeta'(0)$  at  $m^2 a^2 \rightarrow \infty$  are presented.

	$L$	$y(L)$	$c(L)$
$\phi$	0	$\sqrt{m^2 a^2 - 9/4}$	$\frac{1}{12}m^4 a^4 - \frac{7}{18}m^2 a^2 + \frac{29}{64}$
$\psi$	1/2	$ ma $	$\frac{1}{6}m^4 a^4 + \frac{1}{18}m^2 a^2$
$A^\mu$	1	$\sqrt{m^2 a^2 - 13/4}$	$\frac{1}{4}m^4 a^4 - \frac{7}{6}m^2 a^2 + \frac{221}{192}$
$\psi^\mu$	3/2	$ ma $	$\frac{1}{3}m^4 a^4 + \frac{13}{9}m^2 a^2$
$h^{\mu\nu}$	2	$\sqrt{m^2 a^2 - 17/4}$	$\frac{5}{12}m^4 a^4 - \frac{5}{18}m^2 a^2 - \frac{3655}{576}$

Table 4: Some constants appearing in Eq. (87)

	$180 \times \zeta'(0) \text{ at } m^2 a^2 \rightarrow \infty$
$\phi$	$-15m^4 a^4 (\ln m^2 a^2 - 3/2) + 60m^2 a^2 (\ln m^2 a^2 - 1) - 58 \ln m^2 a^2 + \dots$
$\psi$	$-30m^4 a^4 (\ln m^2 a^2 - 3/2) - 60m^2 a^2 (\ln m^2 a^2 - 1) - 11 \ln m^2 a^2 + \dots$
$A^\mu$	$-45m^4 a^4 (\ln m^2 a^2 - 3/2) + 90m^2 a^2 (\ln m^2 a^2 - 1) + 21 \ln m^2 a^2 + \dots$
$\psi^\mu$	$-60m^4 a^4 (\ln m^2 a^2 - 3/2) - 480m^2 a^2 (\ln m^2 a^2 - 1) - 802 \ln m^2 a^2 + \dots$
$h^{\mu\nu}$	$75m^4 a^4 (\ln m^2 a^2 - 3/2) - 300m^2 a^2 (\ln m^2 a^2 - 1) + 10 \ln m^2 a^2 + \dots$

Table 5: The asymptotic forms of a derivative of the generalized zeta function at  $s = 0$

## References

- [1] I. Affleck and M. Dine, *Nucl. Phys.* **B249** 361 (1985).
- [2] J. Wess and J. Bagger, *Supersymmetry and Supergravity* (Princeton Series in Physics, 1983).
- [3] M.T. Grisaru, M. Roček, and W. Siegel, *Nucl. Phys.* **B159** 429 (1979).
- [4] S. Coleman and E. Weinberg, *Phys. Rev.* **D7** 1888 (1973).
- [5] For recent works, see A.L. Berkin, *Phys. Rev.* **D46** 1551 (1992); G. Cognola, K. Kirsten and S. Zerbini, *Phys. Rev.* **D48** 790 (1993); K. Kirsten, G. Cognola and L. Vanzo, *Phys. Rev.* **D48** 2813 (1993); A. Bytsenko, K. Kirsten and S.D. Odintsov, *Mod. Phys. Lett.* **A8** 2011 (1993); E. Elizalde, K. Kirsten and S.D. Odintsov, *Phys. Rev.* **D50** 5137 (1994); E. Elizalde and S.D. Odintsov, *Phys. Lett.* **B303** 240 (1993); **B306** 233 (1993); **B321** 199 (1994); **B333** 331 (1994); *Z. Phys.* **C64** 699 (1994); *Phys. Rev.* **D51** 1680 (1995).
- [6] S.D. Odintsov, *Phys. Lett.* **B213** 7 (1988).
- [7] For a review, see I. Buchbinder, S.D. Odintsov and I. Shapiro, *Effective Action in Quantum Gravity* (IOP publishing, Bristol, 1992) and reference therein.
- [8] N.D. Birrell and P.C.W. Davies, *Quantum fields in curved space* (Cambridge University Press, 1982).
- [9] B.W. Keck, *J. Phys.* **A8** 1819 (1975); B. Zumino, *Nucl. Phys.* **B127** 189 (1977); P. Breitenlohner and D.Z. Freedman, *Ann. Phys. (N.Y.)* **144** 249 (1982); *Phys. Lett.* **B115** 197 (1982); C.J.C. Burges, D.Z. Freedman, S. Davis and G.W. Gibbons, *Ann. Phys. (N.Y.)* **167** 285 (1986); S. Bellucci, *Fortschr. Phys.* **40** 393 (1992) and reference therein.
- [10] K. Symanzik, *Cargèse Lectures in Physics Vol. 5*, ed. D. Bessis (Gordon and Breach, 1972) 179.
- [11] T.S. Bunch and L. Parker, *Phys. Rev.* **20** 2499 (1979).
- [12] C.W. Misner, K.S. Thorne and J.A. Wheeler, *Gravitation* (Freeman, San Francisco, 1973).

- [13] S.J. Gates, Jr., M.T. Grisaru, M. Rocek and W. Siegel, *Superspace: One Thousand and One Lessons in Supersymmetry* (Benjamin-Cummings, Reading, Mass., 1983).
- [14] K. Tobe, *Master Thesis* in Japanese, (Tohoku University, 1994) and reference therein.
- [15] H. Suzuki and M. Tanaka, *Phys. Rev.* **D49** 6692 (1994).
- [16] M. Tanaka, *Prog. Theor. Phys.* **92** 1105 (1994).
- [17] A.S. Schwarz, *Commun. Math. Phys.* **64** 233 (1979).
- [18] S.W. Hawking, *Commun. Math. Phys.* **55** 133 (1977).
- [19] B. Allen, *Nucl. Phys.* **B226** 228 (1983).
- [20] See for example, M. Abramowitz and I. Stegun, *Handbook of Mathematical Functions* (Dover, 1972).